Eur. Phys. J. B **50**, 231–236 (2006)
DOI: 10.1140/epjb/e2006-00141-x

THE EUROPEAN PHYSICAL JOURNAL B

Network traffic behaviour near phase transition point

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Received 2 November 2005
Published online 12 April 2006 –

Abstract. We explore packet traffic dynamics in a data network model near phase transition point from free flow to congestion. The model of data network is an abstraction of the Network Layer of the OSI (Open Systems Interconnect) Reference Model of packet switching networks. The Network Layer is responsible for routing packets across the network from their sources to their destinations and for control of congestion in data networks. Using the model we investigate spatio-temporal packets traffic dynamics near the phase transition point for various network connection topologies, and static and adaptive routing algorithms. We present selected simulation results and analyze them.

PACS. 89.20.Ff Computer science and technology – 89.20.Hh World Wide Web, Internet – 89.75.-k Complex systems – 05.65.+b Self-organized systems

1 Introduction

A *Packet Switching Network* (PSN) is a data communication network consisting of a number of nodes (i.e., routers and hosts) that are interconnected by communication links. Its purpose is to transmit messages from their sources to their destinations. In a PSN each message is partitioned into smaller units of information called *packets* that are capsules carrying the information payload. Packets are transmitted across a network from source to destination via different routes. Upon arrival of all packets to the destination, the message is rebuilt. PSNs owe their name to the fact that packets are individually switched among routers. Some examples of PSNs are the Internet, wide area networks (WANs), local area networks (LANs), wireless communication systems, ad-hoc networks, and sensors networks. There is vast engineering literature devoted to PSNs, see [1–3] and the references therein. Wired PSNs are described by the ISO (International Standard Organization) OSI (Open Systems Interconnect) 7 layers Reference Model [2,3]. We focus on the Network Layer because it plays the most important role in the packet traffic dynamics of the network. It is responsible for routing packets from their sources to their destinations and for control of congestion. The dynamics of flow of packets in PSN can be very complex. It is not well understood how these dynamics depend on network connection topology coupled with various routing algorithms. Understanding of packet flow dynamics is important for further evolution of PSNs, improvements in their design, operation and defence strategies. Some aspects of these dynamics can be studied using models of data networks. With different goals in mind and at various levels of abstractions, models of PSNs have been proposed and applied (e.g., [1,4–13] and the articles therein). In our work we explore packet traffic dynamics near phase transition point from free flow to congestion. We study how this dynamics is affected by the coupling of network connection topology with routing algorithms. We consider static and adaptive routing. Continuing our work of [10,14,15], we use an abstraction of the Network Layer (see [9,10]) and a C++ simulator, called Netzwerk-1 (see [10,16]) that was developed. Like in real networks our model is concerned primarily with packets and their routing; it is scalable, distributed in space, and time discrete. It avoids the overhead of protocol details present in simulators designed with different goals in mind.

2 Packet switching network model description

In our PSN model (see [9,10]) each message consists only of one packet carrying only the following information: time of creation, destination address, and number of hops taken. Each node can perform the functions of a *host* and a *router*. Packets are created randomly and independently at each node. The probability λ with which packets are created is called *source load*. Each node maintains one

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incoming and one outgoing queue to store packets. The outgoing queues are of unlimited length and operate in a first-in, first-out manner. Each node at each time step routes the packet from the head of its outgoing queue to the next node on its route independently from the other nodes. A discrete time, synchronous and spatially distributed network algorithm implements the creation and routing of packets [9,10].

We view a PSN connection topology as a weighted directed multigraph $\mathcal{L}$ where each node corresponds to a vertex and a pair of parallel edges oriented in opposite directions represents each communication link. We associate a packet transmission cost to each directed edge, thus parallel edges do not necessarily share the same cost. Here we consider network connection topologies of the type $\mathcal{L} = \mathcal{L}^p_\Box(L,l)$, that is, isomorphic to a two-dimensional periodic square lattice with L nodes in the horizontal and vertical directions and l additional links added to this square lattice. Notice, that if a sufficient number $l = l_1 + l_2$ of additional links is added to $\mathcal{L} = \mathcal{L}^p_\Box(L,0)$, with l_1 links added in a proper way, then one can obtain a network connection topology of the type $\mathcal{L}^p_\Box(L, l_1 + l_2) = \mathcal{L}^p_\triangle(L, l_2)$, that is isomorphic to a two-dimensional periodic triangular lattice with L nodes in the horizontal and vertical directions and l_2 additional links added to this triangular lattice. All links in the network are static for the duration of each simulation run.

In the PSN model, each packet is transmitted via routers from its source to its destination according to the routing decisions made independently at each router and based on a least-cost criterion. We consider routing decisions based on the *minimum least-cost criterion,* that is, *minimum route distance* or the *minimum route length* depending on the cost assigned to each edge of the graph [1,5]. We consider the following *edge cost functions* called One (ONE), QueueSize (QS), or QueueSizePlusOne ($QSPO$) [9,10]. In each PSN model set-up all edge costs are computed using the same type of edge cost function.

Edge cost function ONE assigns a value of one to all edges in the lattice $\mathcal{L}$. This results in the *minimum hop routing* (minimum route distance) when a least cost routing criterion is applied, because the number of hops taken by each packet between its source and its destination node is minimized. This type of routing is called *static routing*, since the values assigned to each edge do not change during the course of a simulation. The edge cost function QS assigns to each edge in the lattice $\mathcal{L}$ a value equal to the length of the outgoing queue at the node from which the edge originates. When this edge cost function is used a packet traversing a network will travel from its current node to the next node along an edge belonging to a path with the least total number of packets in transit between its current location and its destination at this time. The edge cost function $QSPO$ assigns a value that is the sum of a constant one plus the length of the outgoing queue at the node from which the edge originates. This cost function combines the features of the previous two functions. The costs QS and $QSPO$ are derived for each edge from the router's load. Since the routing decisions made using QS or $QSPO$ edge cost function rely on the current state of the network simulation they imply *adaptive* or *dynamic routing.* In these types of routing packets have the ability to avoid congested nodes during a network simulation.

Our PSN model uses *full-table routing*, that is, each node maintains a *routing table* of least path cost estimates from itself to every other node in the network. Because in a PSN model using edge cost function ONE the costs of edges are static during a simulation, the routing tables are calculated only at the beginning, they are not updated, and the cost estimates are the precise least-costs [10]. When the edge cost function QS or $QSPO$ is used, the routing tables are updated at each time step using a distributed version of Bellman-Ford least-cost algorithm [5]. In both cases the path costs stored in the routing tables are only estimates of the actual least path costs across the network because only local information is exchanged and updated at each time step. In our PSN model time is discrete and we observe its state at the discrete time points $k = 0, 1, 2, \ldots, T$, where T is the final simulation time. At time $k = 0$, the set-up of PSN model is initialised with empty queues and the routing tables are computed using the centralised Bellman-Ford least-cost algorithm [5].

The discrete time, synchronous and distributed in space PSN model algorithm consists of the sequence of five operations advancing the simulation time from k to $k+1$. These operations are: (1) *Update routing tables*; (2) *Create and route packets*; (3) *Process incoming queue*; (4) *Evaluate network state*; (5) *Update simulation time.* This algorithm is described in [9,10].

3 Packet traffic behaviour near phase transition point

The phase transition from congestion-free to congested state was observed in empirical studies of PSNs [17] and motivated further research (e.g., [13] and the articles therein). Understanding the dynamics of this transition has practical implications leading to more efficient designs of PSNs. In our PSN model, for a particular family of network set-ups, which differ only in the value of the *source load* λ, values λ for which packet traffic is congestion-free are called *sub-critical* while values for which traffic is congested are called *super-critical.* The *critical source load* λ_c, that is, the source load at the phase transition point, is the largest sub-critical source load. The explanation how we estimate its value in simulations is given in [10].

Here, we explore how spatio-temporal dynamics of packet traffic in our PSN model is affected by the network connection topology and edge cost function type for source loads close to the phase transition point of each PSN model set-up. As a case study we consider network connection topologies of the type $\mathcal{L}^p_\Box(L,l)$ and $\mathcal{L}^p_\triangle(L,l)$, where $L = 16$ and $l = 0$ or 1. We say that $\mathcal{L}^p_\Box(L,0)$ and $\mathcal{L}^p_\triangle(L,0)$, that is, the regular network connection topologies are *undecorated* and we say that they are *decorated* if an extra link is added to each of them, that is, they are

Table 1. Critical source load values.

	$\mathcal{L}^p_{\square}(16,0)$	$\mathcal{L}^p_{\square}(16,1)$	$\mathcal{L}^p_{\triangle}(16,0)$	$\mathcal{L}^p_{\triangle}(16,1)$
ONE	0.115	0.020	0.140	0.030
QS	0.120	0.125	0.155	0.160
QSPO	0.120	0.125	0.155	0.160

of the type $\mathcal{L}^p_{\square}(L,1)$ and $\mathcal{L}^p_{\triangle}(L,1)$. We use the following convention if we want to specify additionally what type of an edge cost function our PSN model is using, namely, $\mathcal{L}^p_{\square}(16,l,ecf)$ and $\mathcal{L}^p_{\triangle}(16,l,ecf)$, where $l = 0$ *or* 1, and $ecf = ONE$, or QS, or $QSPO$.

For the considered network connection topologies the estimated critical source load values λ_c are provided in Table 1. An extensive study about how the values of λ_c depend on the network connection topology, the edge cost function type, and the mode of routing table update in the considered PSN model is presented in [10] and reference therein.

Figures of Tables 2–4 display spatial distribution of outgoing queue sizes at nodes for various PSN model set-ups. The x- and y- axis coordinates of each figure denote the positions of switching nodes and z-axis denotes the number of packets in the outgoing queue of the node located at that (x, y) position. The header of each column of each table and the parameter shown under each figure uniquely define the PSN model set-up, the simulation time, the values of critical source load λ_c and super-critical source load $\lambda_{\sup c} = \lambda_c + 0.005$.

From Tables 2 and 3 we observe that the qualitative behaviour of spatial distribution of outgoing queue sizes is very similar for PSN model set-up $\mathcal{L}^p_{\square}(16,0,ONE)$ and $\mathcal{L}^p_{\triangle}(16,0,ONE)$ when the source loads are λ_c and $\lambda_{\sup c}$, respectively. In each case queue sizes are randomly distributed with large fluctuations. Increase in source load results in substantial increase of queue sizes and fluctuations among them (notice, the figures use different scales on z-axis). The discussed behaviours for λ_c and $\lambda_{\sup c}$ are typically observed for other sub-critical source load values and super-critical ones, respectively, in these types of PSN model set-ups.

From Tables 2 and 3 we observe also a random distribution of queue sizes in each PSN model set-up $\mathcal{L}^p_{\square}(16,0,ecf)$ and $\mathcal{L}^p_{\triangle}(16,0,ecf)$, where $ecf = QS$, $QSPO$, and the source loads are λ_c. But, the magnitudes of the fluctuations are smaller than those in the case of PSN model set-ups using the edge cost function ONE. For the other sub-critical source load values in the PSN model set-ups $\mathcal{L}^p_{\square}(16,0,ecf)$ and $\mathcal{L}^p_{\triangle}(16,0,ecf)$, where $ecf = QS$ or $QSPO$, queue size distributions share the same characteristics. However, we observe a big qualitative difference between the distribution of queue sizes of the PSN model set-up $\mathcal{L}^p_{\square}(16,0,QSPO)$ and those of the PSN model set-ups $\mathcal{L}^p_{\square}(16,0,QS)$, $\mathcal{L}^p_{\triangle}(16,0,QS)$, and $\mathcal{L}^p_{\triangle}(16,0,QSPO)$, when their respective $\lambda_{\sup c}$ source loads are used, see Tables 2 and 3.

In each of the PSN model set-up $\mathcal{L}^p_{\square}(16,0,QSPO)$ and $\mathcal{L}^p_{\square}(16,0,QS)$ for $\lambda_{\sup c}$ (i.e., in the congested state of the network) we observe spatio-temporal self-organization in the sizes of the outgoing queues and the emergence of a pattern of peaks and valleys, see Tables 2 and 4. However, the time scale on which the pattern emerges in the PSN model set-up $\mathcal{L}^p_{\square}(16,0,QS)$ is much longer than the time scale of the PSN model set-up $\mathcal{L}^p_{\square}(16,0,QSPO)$. Also, the difference between sizes of the neighbouring peaks and valleys increases much faster with time in the PSN model set-up $\mathcal{L}^p_{\square}(16,0,QSPO)$ than the one of $\mathcal{L}^p_{\square}(16,0,QS)$. This could imply that, when the edge cost function $QSPO$ is used, the cost component ONE is responsible for the observed qualitative differences in the evolution of the spatio-temporal packet traffic dynamics between the network models under consideration. A similar behaviour was observed for values of L other than 16 and super-critical source load values other than $\lambda_{\sup c}$. In the congested state as the number of packets increases the pattern of peaks and valleys emerges in spite of the adaptive routing attempts to distribute evenly the packets among the queues.

Looking at Tables 3 and 4 we see that adding an extra link to the connection topology of the PSN model set-up $\mathcal{L}^p_{\square}(16,0,QS)$ speeds up the "peak-valley" pattern emergence in a congested state. This is also true when the extra link has other position and/or length and L is different than 16, see [10,15]. The same phenomenon was observed in the congested states when edge cost function $QSPO$ was used, see [10,14].

In spite the fact that the considered adaptive routings try to distribute packets evenly among the network nodes the effect of an extra link on the packet traffic dynamics is much stronger. It provides a "short-cut in communication" among distant nodes. This speeds up the emergence of the "peak-valley" pattern in the network models with square lattice connection topologies and edge cost functions QS and $QSPO$. This is also true when instead of one extra link a relatively small number of extra links is added, see [10]. If a larger number of extra links is added the pattern of peaks-valleys does not emerge, see Tables 3, 4 and [14]. We observe rather small differences among the outgoing queue sizes in congested states of the PSN model set-ups $\mathcal{L}^p_{\triangle}(16,0,QS)$ and $\mathcal{L}^p_{\triangle}(16,0,QSPO)$. Also, from Table 4 we notice that when an extra link is added to the periodic triangular network connection topology, these differences become even smaller, except of the two nodes to which the extra link is attached. These nodes attract much larger numbers of packets than other nodes resulting in local congestion. Qualitatively similar behaviour was observed for the PSN model set-up $\mathcal{L}^p_{\triangle}(16,1,QSPO)$ in its congested state.

4 Conclusions

We investigated the effects of coupling of network connection topology with, respectively, static and adaptive routings on spatio-temporal packet traffic dynamics near phase transition point from free flow to congested state in

Table 2. For critical and super-critical source loads spatial distribution of outgoing queue sizes in the PSN model set-ups defined in the table header.

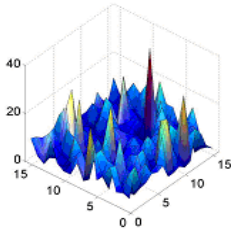
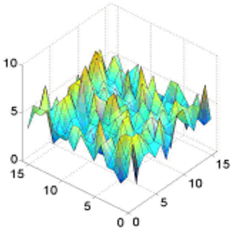
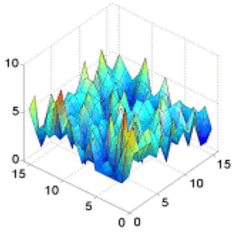
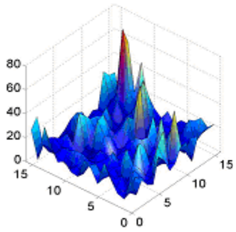
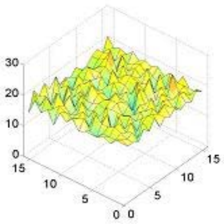
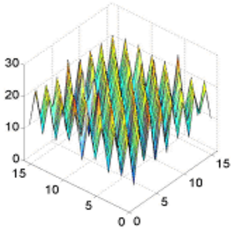
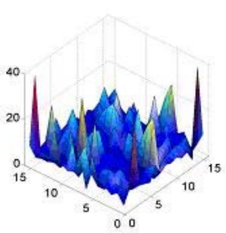
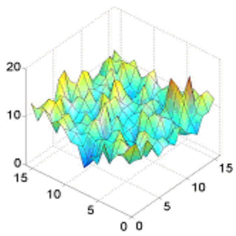
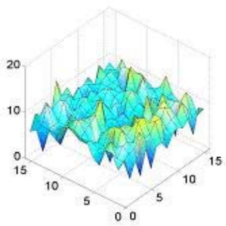
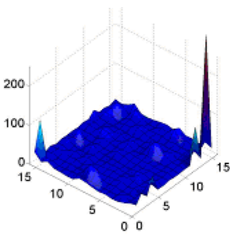
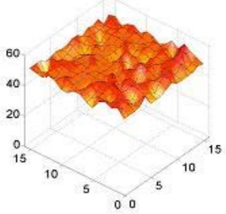
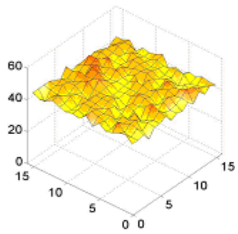
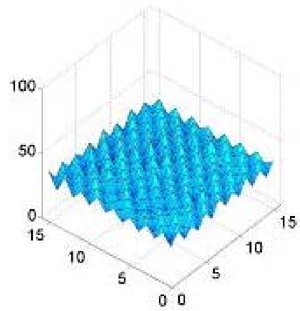
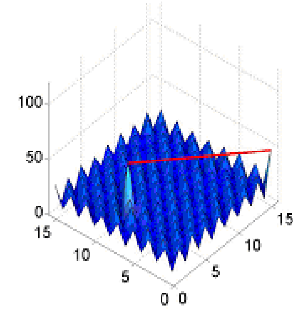
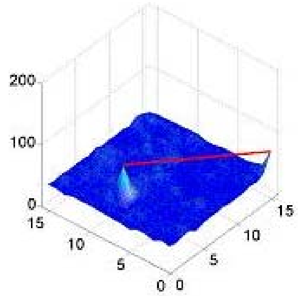
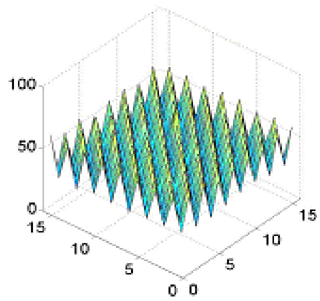
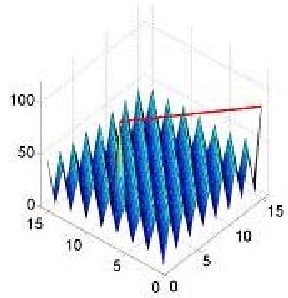
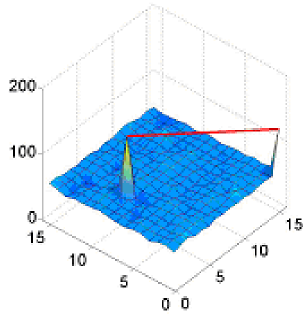
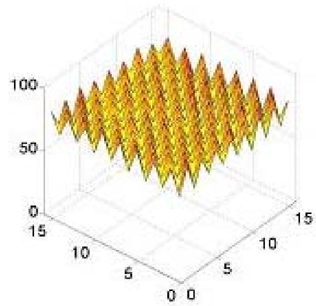
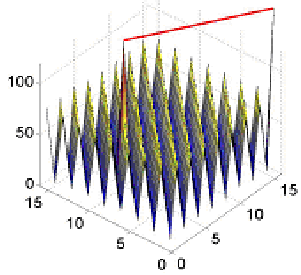
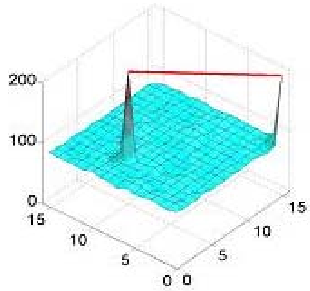
$\mathcal{L}^p_{\square}(16, 0, ONE), k = 8000$	$\mathcal{L}^p_{\square}(16, 0, QS), k = 8000$	$\mathcal{L}^p_{\square}(16, 0, QSPO), k = 8000$
		
$\lambda_c = 0.115$	$\lambda_c = 0.120$	$\lambda_c = 0.120$
		
$\lambda_{\sup c} = 0.120$	$\lambda_{\sup c} = 0.125$	$\lambda_{\sup c} = 0.125$

Table 3. For critical and super-critical source loads spatial distribution of outgoing queue sizes in the PSN model set-ups defined in the table header.

$\mathcal{L}^p_{\triangle}(16, 0, ONE), k = 8000$	$\mathcal{L}^p_{\triangle}(16, 0, QS), k = 8000$	$\mathcal{L}^p_{\triangle}(16, 0, QSPO), k = 8000$
		
$\lambda_c = 0.140$	$\lambda_c = 0.155$	$\lambda_c = 0.155$
		
$\lambda_{\sup c} = 0.145$	$\lambda_{\sup c} = 0.160$	$\lambda_{\sup c} = 0.160$

our PSN model. We observed that for adaptive routings and periodic square lattice network connection topologies patterns of "peaks-valleys" emerge in the distributions of outgoing queue sizes when the network models are in their congested states. The emergence of this type of synchronization is accelerated by addition of an extra link and is destroyed by addition of many links. Synchronization in other types of networks are discussed in [18,19] and references therein. For our PSN model with adaptive routings and periodic triangular lattice connection topologies we observed that packet traffic is much more evenly distributed. We demonstrated that dynamics of the PSN model depend on the coupling between network connection topology and routing. The presented work is continuation of our research in [10,14,15] and contributes to the study of dynamics of complex networks.

Table 4. Spatial distribution of outgoing queue sizes at various times k in the PSN model set-ups defined in the table header.

$\mathcal{L}^p_{\square}(16, 0, QS), \lambda_{\sup c} = 0.125$	$\mathcal{L}^p_{\square}(16, 1, QS), \lambda_{\sup c} = 0.130$	$\mathcal{L}^p_{\triangle}(16, 1, QS), \lambda_{\sup c} = 0.165$
 $k = 20000$	 $k = 2000$	 $k = 2000$
 $k = 40000$	 $k = 4000$	 $k = 4000$
 $k = 80000$	 $k = 8000$	 $k = 8000$

A.T.L. acknowledges partial financial support from Sharcnet and NSERC of Canada. X.T. acknowledges partial financial support from Sharcnet and the University of Guelph. The authors thank B. Di Stefano, A. Gerisch and K. Maxie for helpful discussions.

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